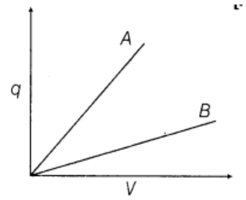
Capacitance

1 Mark Questions



Ans. Line **B** corresponds to C-, because slope (q/v) of **B** is less than slope of A.

2.A capacitor has been charged by a DC source. What are the magnitude of conduction and displacement current when it is fully charged?[Delhi 2013]

Ans.

Electric flux through capacitor's plates,

$$\phi_E = \frac{q}{\varepsilon_0}$$

Here, q = charge on capacitor

Displacement current,

$$I_D = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d\left(\frac{q}{\varepsilon_0}\right)}{dt} = 0$$

Conduction current, $I = C\frac{dV}{dt} = 0$, as voltage becomes constant when the capacitor becomes fully charged. **(1)**

3.Distinguish between a dielectric and a conductor. [Delhi 2012]

Ans.Dielectrics are non-conductors and do not have free electrons at all. While conductor has free electrons in its any volume which makes it able to pass the electricity through it.

4.Define the dielectric constant of a medium. What is its unit? [Delhi 2011c]

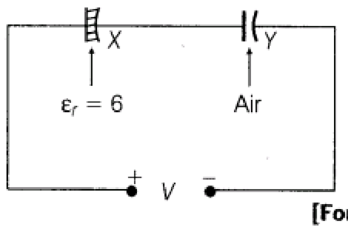
Ans. Dielectric When a dielectric slab is introduced between the plates of charged capacitor or in the region of electric field, an electric field E_P induces inside the dielectric due to induced charge on dielectric in a direction opposite to the direction of applied external electric field. Hence, net electric field inside the dielectric gets reduced to $£_0 - £_P$, where $£_0$ is external electric field. The ratio of applied external electric field and reduced electric field is known as dielectric constant K of dielectric medium, i.e.

$$K = \frac{E_0}{E_0 - E_P}$$

5.A metal plate is introduced between the plates of a charged parallel plate capacitor. What is its effect on the capacitance of the capacitor?[Foreign 2009]

Ans. If a metal plate is introduced between the plates of a charged parallel plate capacitor. The capacitance of parallel plate capacitor will becomes infinite.

6.In the figure given below X, Y represent parallel plate capacitors having the same area of plates and the same distance of separation between them. What is the relation between the energies stored in the capacitors?



[Foreign 2008]

Ans.

Charge on each capacitor connected in series combination is same. Also, two capacitors having same plate area and in equal separation have equal capacitance in air or vacuum.

$\because \quad C_X = 6C_Y$

Charge, q is same on both capacitors as connected in series combination.

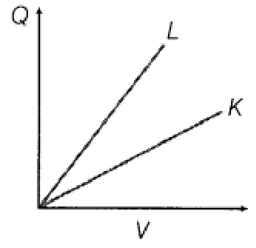
Energy stored in a capacitor is given by

$$U = \frac{q^2}{2C} \Rightarrow U \propto \frac{1}{C}$$

$$\therefore \quad \frac{U_X}{U_Y} = \frac{C_Y}{C_X} = \frac{1}{6}$$

$$U_X = \frac{1}{6} U_Y$$ **(1)**

7.The following graph shows the variation of charge Q with voltage V for two capacitors K and L In which capacitor is more electrostatic energy stored?



[HOTS, Foreign 2008]

Ans.

The slope of Q-V graph of capacitor gives capacitance. Therefore, capacitor L have greater capacitance. Since, two capacitors are given same voltage V, therefore energy stored in a capacitor will be given by formula.

$$U = \frac{1}{2}CV^2 \text{ and for same } V$$

$$U \propto C$$

$\because \quad C_L > C_K$

$\Rightarrow \quad U_L > U_K$

$\Rightarrow$ Capacitor L will store more electrostatic energy. **(1)**

8.Define dielectric strength of a dielectric. [Delhi 2008 C]

Ans. The dielectric strength of a dielectric is the maximum value of applied electric field required to just breakdown of the dielectric material.

2 Marks Questions

9.A parallel plate capacitor of capacitance C is charged to a potential V. It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that stored initially in the single capacitor.[All India 2014]

Ans.

Let q be the charge on the charged capacitor.

∴ Energy stored in it is given by

$$U = \frac{q^2}{2C}$$

When another uncharged similar capacitor is connected, then the net capacitance of the system is given by

$$C' = 2C \quad \textbf{(1)}$$

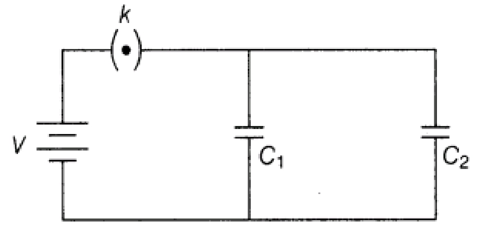
The charge on the system remains constant. So, the energy stored in the system is given by

$$U' = \frac{q^2}{2C'} = \frac{q^2}{4C} \quad [\because C' = 2C]$$

Thus, the required ratio is given by

$$\frac{U'}{U} = \frac{q^2/4C}{q^2/2C} = \frac{1}{2}$$

10.Two parallel plate capacitors of capacitances C_x and C_2 such that $C_x = 2C_2$ are connected across a battery of V volt as shown in the figure Initially, the key (k) is kept closed to fully charge the capacitors.The key is now thrown open and a dielectric slab of dielectric constant K is inserted in the two capacitors to completely fill the gap between the plates. Find the ratio of (i) the net capacitance and (ii) the energies stored in the combination before and after the introduction of the dielectric slab.[Delhi 2014 C]



Ans.

(*i*) Given, $C_1 = 2C_2$ ...(i)

Net capacitance before filling the gap with dieletric slab is given by

$C_{\text{initial}} = C_1 + C_2$ [from Eq. (i)]

$C_{\text{initial}} = 2C_2 + C_2 = 3C_2$ ...(ii)

Net capacitance after filling the gap with dieletric slab of electric constant K

$C_{\text{initial}} = KC_1 + KC_2 = K(C_1 + C_2)$ [from Eq. (ii)]

$C_{\text{final}} = 3KC_2$ ...(iii)

Ratio of net capacitance is given by

$\frac{C_{\text{initial}}}{C_{\text{final}}} = \frac{3C_2}{3KC_2} = \frac{1}{K}$ [from Eqs. (ii) and (iii)]

(1)

(*ii*) Energy stored in the combination before introduction of dielectric slab

$U_{\text{initial}} = \frac{Q^2}{3C_2}$ ...(iv)

Energy stored in the combination after introduction of dielectric slab.

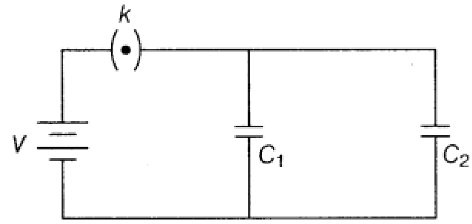
$U_{\text{final}} = \frac{Q^2}{3KC_2}$ ... (v)

Ratio of energies stored

$\frac{U_{\text{initial}}}{U_{\text{final}}} = \frac{K}{1}$ [from Eqs. (iv) and (v)]

(1)

11.Two parallel plate capacitors of capacitances Qand C_2 such that q $=C_2/2$ are connected across a battery of V volts as shown in the figure. Initially, the key (k) is kept closed to fully charge the capacitors.The key is now thrown open and a dielectric slab of dielectric constant K is inserted in the two capacitors to completely fill the gap between the plates. Find the ratio of (i) the net capacitance and (ii) the energies stored in the combination before and after the introduction of the dielectric slab.[Delhi 2014 C]



Ans.

(*i*) Given, $C_1 = 2C_2$ ...(i)

Net capacitance before filling the gap with dieletric slab is given by

$C_{initial} = C_1 + C_2$ [from Eq. (i)]

$C_{initial} = 2C_2 + C_2 = 3C_2$ ...(ii)

Net capacitance after filling the gap with dieletric slab of electric constant K

$C_{initial} = KC_1 + KC_2 = K(C_1 + C_2)$ [from Eq. (ii)]

$C_{final} = 3KC_2$ ...(iii)

Ratio of net capacitance is given by

$$\frac{C_{initial}}{C_{final}} = \frac{3C_2}{3KC_2} = \frac{1}{K}$$ [from Eqs. (ii) and (iii)] **(1)**

(*ii*) Energy stored in the combination before introduction of dielectric slab

$$U_{initial} = \frac{Q^2}{3C_2}$$ ...(iv)

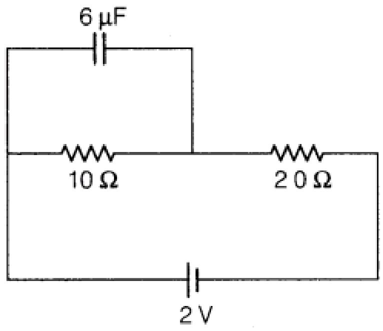
Energy stored in the combination after introduction of dielectric slab.

$$U_{final} = \frac{Q^2}{3KC_2}$$ ... (v)

Ratio of energies stored

$$\frac{U_{initial}}{U_{final}} = \frac{K}{1}$$ [from Eqs. (iv) and (v)] **(1)**

12.Find the charge on the capacitor as Shown in the circuit. [Foreign 2014]



Ans.

Total current through the circuit is given by $I = V / R$

Here, $V = 2$ V

$$R = (10 + 20)\,\Omega = 30\,\Omega$$

$$\therefore \quad I = \frac{2}{30} = \frac{1}{15}\text{ A}$$

Voltage across 10 Ω resistor

$$= I(10) = 10/15 = \frac{2}{3}\text{ V}$$

Charge on the capacitor is given by $Q = CV = (6 \times 10^{-6}) \times 2/3 = 4\mu\text{ C}$ **(1)**

13.A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor, but has the thickness d/2, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor. [Delhi 2013]

Ans.

Initially, when there is a vacuum between the two plates, then capacitance of the plate is $C_0 = \frac{\varepsilon_0 A}{d}$, where A is the area of parallel plates.

Suppose that the capacitor is connected to a battery, an electric field E_0 is produced. Now, if we insert the dielectric slab of thickness $t = d/2$, the electric field reduces to E.

Now, the gap between plates is divided in two parts, for distance t there is electric field E and for the remaining distance $(d - t)$ the electric field is E_0. **(1)**

If V be the potential difference between the plates of the capacitor, then $V = Et + E_0(d - t)$

$$V = \frac{Ed}{2} + \frac{E_0 d}{2} = \frac{d}{2}(E + E_0) \quad \left[\because t = \frac{d}{2}\right]$$

$$\Rightarrow \quad V = \frac{d}{2}\left(\frac{E_0}{K} + E_0\right)$$

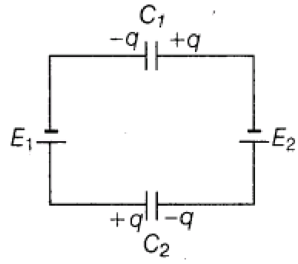
$$= \frac{dE_0}{2K}(K+1) \quad \left[\text{As, } \frac{E_0}{E} = K\right]$$

Now, $E_0 = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A}$

$$\Rightarrow \quad V = \frac{d}{2K} \cdot \frac{q}{\varepsilon_0 A}(K+1)$$

We know that, $C = \frac{q}{V} = \frac{2K\varepsilon_0 A}{d(K+1)}$ **(1)**

14.Determine the potential difference across the plates of the capacitor C_x of the network shown in the figure, (assume, E_2 >E1) [All India 2013]

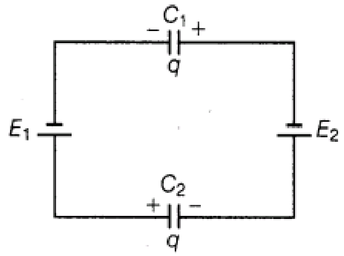


[All India 2013]

Ans.

Potential difference, $\frac{-q}{C_1} + E_1 - \frac{q}{C_2} - E_2 = 0$

or $$\frac{q}{C_1} + \frac{q}{C_2} = E_1 - E_2$$ **(1)**



Now, $V_1 = \frac{-q}{C_1}$ and $V_2 = \frac{+q}{C_2}$ **(1)**

15. Net capacitance of three identical capacitors in series is 1 μF. What will be their net capacitance if connected in parallel?

Find the ratio of energy stored in the two configurations if they are both connected to the same source.

[All India 2011]

Ans.

If n identical capacitors, each of capacitance C are connected in series combination give equivalent capacitance,

$$C_s = \frac{C}{n}$$

and when connected in parallel combination then equivalent capacitance,

$$C_p = nC$$

Also, for same voltage, energy stored in the capacitor is given by

$$U = \frac{1}{2}CV^2 \qquad \text{[for constant]}$$

$$U \propto C$$

$$C_s = 1\,\mu F \qquad [\because n = 3]$$

In series combination, $C_s = \frac{C}{n}$

In parallel combination,

$\Rightarrow \quad C_p = nC$

According to the problem,

$$C = nC_s = 3 \times 1\,\mu F = 3\,\mu F$$

For each capacitor,

In parallel combination,

$$C_p = nC = 3 \times 3 = 9\,\mu F$$

$$C_p = 9\,\mu F \qquad \textbf{(1)}$$

For same voltage, $U \propto C \Rightarrow \frac{U_s}{U_p} = \frac{C_s}{C_p}$

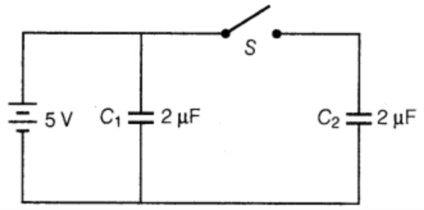
$$= \frac{C/n}{nC} = \frac{1}{n^2}$$

$$\Rightarrow \quad \frac{1}{(3)^2} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{U_s}{U_p} = \frac{1}{9}$$

or $\quad U_s : U_p = 1:9$ **(1)**

16. Figure shows two identical capacitors C_1 and C_2, each of $2\,\mu F$ capacitance, connected to a battery of 5 V. Initially, switch S is closed. After sometime, S is left open and dielectric slabs of dielectric constant $K = 5$ are inserted to fill completely the space between the plates of the two capacitors. How will the (*i*) charge and (*ii*) potential difference between the plates of the capacitors be affected after the slabs are inserted?



Ans.

When a dielectric medium of dielectric constant K is introduced,

(i) in an isolated (not connected with battery) capacitor, then total charge on capacitor remains same.

(ii) in a capacitor connected with battery, then potential difference across the capacitor remains same as that of potential difference across battery.

Two identical capacitors C_1 and C_2 get fully charged with 5 V battery initially.

So, the charge and potential difference on both capacitors becomes

$$q = CV$$
$$= 2 \times 10^{-6} \times 5 \text{ V} = 10 \text{ } \mu\text{C}$$

and $V = 5 \text{ V}$

On introduction of dielectric medium of $K = 5$.

For C_1 (continue to be connected with battery) potential difference of C_1, $(V') = 5$ V

Capacitance of $C_1' = KC = 5 \times 2 \text{ } \mu\text{F} = 10 \text{ } \mu\text{F}$

Charge, $q' = C'V' = (10 \text{ } \mu\text{F})(5 \text{ V}) = 50 \text{ } \mu\text{C}$ **(1)**

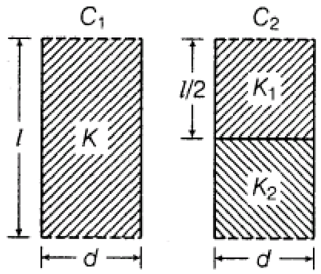
For C_2 (disconnected with battery)

Charge, $q' = q = 10 \text{ } \mu\text{C}$

Potential difference,

$$V' = \frac{V}{K} = \frac{5}{5} = 1 \text{ V}$$ **(1)**

17.Two identical parallel plate (air) capacitors C_x and C_2 have capacitance C each. The space between their plates is now filled with dielectrics as shown in the figure. If the two capacitors still have equal capacitance, they obtain the relation between dielectric constants K, K_x andK_2.[Foreign 2011]



Ans.

The capacity of condenser is proportional to the area and inversely proportional to the distance between its plates. If a medium of dielectric constant K is filled in the space between the plates, its capacity becomes K times the capacity when there is air between the plates.

After inserting the dielectric medium, let their capacitances become C_1' and C_2'.

For C_1 $\quad C_1' = KC \quad$ ...(i) **(1/2)**

For C_2 $\quad C_2' = \frac{K_1\varepsilon_0 (A/2)}{d} + \frac{K_2\varepsilon_0 A/2}{d}$

C_2 acts as if two capacitors each of area $A/2$ and separation d are connected in parallel combination.

$$C_2' = \frac{\varepsilon_0 A}{d}\left(\frac{K_1}{2} + \frac{K_2}{2}\right)$$

$$C_2' = C\left(\frac{K_1 + K_2}{2}\right) \quad \text{...(ii)}$$

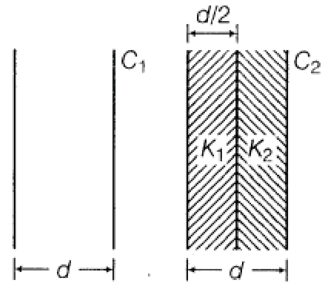
$$\left[\because C = \frac{\varepsilon_0 A}{d}\right]$$ **(1/2)**

According to the problem,

$$C_1' = C_2'$$

$$KC = C\left(\frac{K_1 + K_2}{2}\right) \Rightarrow K = \frac{K_1 + K_2}{2}$$ **(1)**

18.You are given an air filled parallel plate capacitor C_1. The space between its plates is now filled with slabs of dielectric constants K_x and K_2 as shown in figure. Find the capacitance of the capacitor C_2 if area of the plates is A and distance between the plates is d.



[Foreign 2011]

Ans.

After introduction of dielectric medium of dielectric constants K_1 and K_2, capacitor acts as if it consists of two capacitors, each having plates of area A and separation $\frac{d}{2}$ connected in series combination for

$$C_1 = \frac{\varepsilon_0 A}{d} \quad \text{...(i)}$$

$$\frac{1}{C_2} = \frac{1}{\left(\frac{K_1 \varepsilon_0 A}{d/2}\right)} + \frac{1}{\left(\frac{K_2 \varepsilon_0 A}{d/2}\right)} \quad \textbf{(1)}$$

$$\frac{1}{C_2} = \frac{1}{\left(\frac{\varepsilon_0 A}{d}\right)}\left(\frac{1}{2K_1} + \frac{1}{2K_2}\right)$$

$$\frac{1}{C_2} = \frac{1}{2C_1}\left(\frac{K_2 + K_1}{K_1 K_2}\right)$$

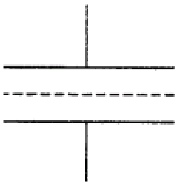
$$C_2 = C_1\left(\frac{2K_1 K_2}{K_1 + K_2}\right) \quad \textbf{(1)}$$

The capacitors will be in series.

19.Figure shows a sheet of aluminium foil of negligible thickness placed between the plates of a capacitor. How will its capacitance be affected if

(i)the foil is electrically insulated?

(ii)the foil is connected to the upper plate with a conducting wire?[Foreign 2011]



Ans.

(*i*) The system will be equivalent to two identical capacitors connected in series combination in which two plates of each capacitor have separation half of the original separation.

Thus, new capacitance of each capacitor

$$C' = 2C \qquad \left[\because C \propto \frac{1}{d}\right]$$

$\because$ C and C' are in series

$$\Rightarrow \quad C_{net} = \frac{2C \times 2C}{2C + 2C} = C$$

$$C_{net} = C$$

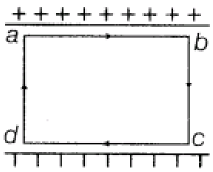
$= $ Original capacitor **(1)**

(*ii*) System reduces to a capacitor whose separation reduces to half of original one.

$\therefore$ New capacitance $C' = 2C$ **(1)**

20.(i)Obtain the expression for the energy stored per unit volume in a charged parallel plate capacitor.

(ii) The electric field inside a parallel plate capacitor is E. Find the amount of work done in moving a charge q over a closed rectangular loop abcda. [Delhi 2014]



Ans.(i) The energy of a charged capacitor is measured by the total work done in charging the capacitor to a given potential.

Let us assume that initially both the plates are uncharged. Now, we have to repeatedly remove small positive charges from one plate and transfer them to other plate.

Now, when an additional small charge (dq) is transferred from one plate to another, the small work done is given by

$$dW = V'dq = \frac{q'}{C}dq$$ **(1)**

[let charge on plate, when dq charge is transferred is q']

The total work done in transferring charge Q is given by

$$W = \int_0^Q \frac{q'}{C}dq = \frac{1}{C}\int_0^Q q'\,dq$$

$$= \frac{1}{C}\left[\frac{(q')^2}{2}\right]_0^Q = \frac{Q^2}{2C}$$

This work is stored as electrostatic potential energy U in the capacitor.

$$U = \frac{Q^2}{2C} = \frac{(CV)^2}{2C} \qquad [\because Q = CV]$$

$$U = \frac{1}{2}CV^2$$ **(1)**

The energy stored per unit volume of space in a capacitor is called energy density.

$$u = \frac{\frac{1}{2}CV^2}{Ad}$$

$$u = \frac{1}{2}\frac{\varepsilon_0 AV^2}{d^2A}$$

Energy density,

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Total energy stored in series combination or parallel combination of capacitors is equal to the sum of energies stored in individual capacitor.

i.e. $U = U_1 + U_2 + U_3 + \ldots$

(*ii*) Due to conservative nature of electric force, the work done in moving a charge in a close path in a uniform electric field is zero. **(1)**

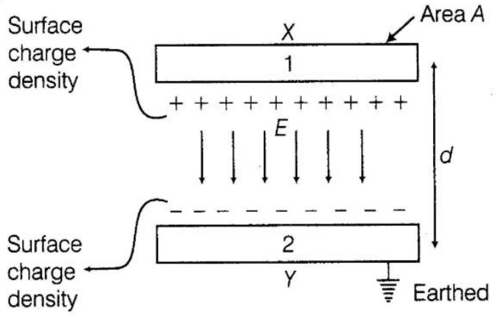
3 Marks Questions

21.(i) Derive the expression for the capacitance of a parallel plate capacitor having plate area A and plate separation d.

(ii) Two charged spherical conductors of radii and 1^ when connected by a conducting plate respectively. Find the ratio of their surface charge densities in terms of their radii. [Delhi 2014]

Ans.

(*i*) Parallel plate capacitor consists of two thin conducting plates each of area A held parallel to each other at a suitable distance d. One of the plates is insulated and other is earthed. There is a vacuum between the plates.



(1)

Suppose, the plate X is given a charge of $+q$ coulomb. By induction, $-q$ coulomb of charge is produced on the inner surface of the plate Y and $+q$ coulomb on the outer surface. Since, the plate Y is connected to the earth, the $+q$ charge on the outer surface flows to the earth. Thus, the plates X and Y have equal and opposite charges. Suppose, the surface density of charge on each plate is σ. We know that the intensity of electric field at a point between two plane parallel sheets of equal and opposite charges is σ / ε_0, where ε_0 is the permittivity of free space.

The intensity of electric field between the plates will be given by, $E = \frac{\sigma}{\varepsilon_0}$

The charge on each plate is q and the area of each plate is A. Thus,

$$\sigma = \frac{q}{A} \text{ and } E = \frac{q}{\varepsilon_0 A} \quad \text{...(i)}$$

Now, let the potential difference between the two plates be V volt. Then, the electric field between the plates is given by

$$E = \frac{V}{d} \text{ or } V = Ed$$

Substituting the value of E from Eq. (i), we get

$$V = \frac{qd}{\varepsilon_0 A}$$

∴ Capacitance of the capacitor is

$$C = \frac{q}{V} = \frac{q}{qd/\varepsilon_0 A} \quad \text{or} \quad C = \frac{\varepsilon_0 A}{d}$$

where, $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

It is clear from this formula that in order to obtain high capacitance

(a) A should be large, i.e. the plates of large area should be taken.

(b) d should be small, i.e the plates should be kept closer to each other. **(1)**

(ii) Surface charge density is given by

$$\sigma = \frac{q}{4\pi R^2}$$

After connecting both the conductors, their potentials will become equal.

$$V_1 = V_2$$

$$\Rightarrow \frac{Kq_1}{R_1} = \frac{Kq_2}{R_2} \quad [\because \text{ for spherical conductors } V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \text{ or } V = \frac{KV}{R}]$$

$$\Rightarrow \frac{q_1}{q_2} = \frac{R_1}{R_2}$$

$$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{q_1/4\pi R_1^2}{q_2/4\pi R_2^2}$$

$$= \frac{q_1}{q_2}\left(\frac{R_2}{R_1}\right)^2 = \frac{R_2}{R_1}$$

22. In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3}\,m^2$ and the separation between the plate is 3 mm.

(i) Calculate the capacitance of the capacitor.

(ii) If this capacitor is connected to 100 V supply, what would be the change on each plate?

(iii) How would charge on the plates be affected if a 3 mm thick mica sheet of $K = 6$ is inserted between the plates while the voltage supply remains connected? **[Foreign 2014]**

Ans.

Given, area of each plate, $A = 6 \times 10^{-3}\,m^2$

Distance between plates d,

$$= 3\,mm = 3 \times 10^{-3}\,m$$

(i) Capacitance of parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d}$$

$$\varepsilon_0 = 8.85 \times 10^{-12}$$

$$C = \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}$$

$$C = 1.77 \times 10^{-11}\,F$$ **(1)**

(ii) Charge on parallel plate capacitor is given by

$$Q = CV$$

Given, $V = 100\,V$

Now,

$$Q = 17.7 \times 10^{-12} \times 100$$

$$Q = 1.77 \times 10^{-10} C$$ ...(i) **(1)**

(iii) Given, $K = 6$

Now, $C' = KC$

$\therefore$ $Q' = KQ$ [From Eq. (i)]

$$Q' = 6 \times 1.77 \times 10^{-10}$$

$$Q' = 10.62 \times 10^{-10} C$$ **[1]**

23. A capacitor of unknown capacitance is connected across a battery of V volt. The charge stored in it is $360\mu C$. When potential across the capacitor is reduced by 120 V, the charge stored in it becomes $120\ \mu C$.

Calculate

(i) the potential V and the unknown capacitance C.

(ii) what will be the charge stored in the capacitor if the voltage applied had increased by 120 V? **[Delhi 2**

Ans.

(*i*) We have initial voltage, $V_1 = V$ volt and charge stored, $Q_1 = 360\,\mu C$.

$$Q_1 = CV_1 \quad ...(i)$$

Charged potential, $V_2 = V - 120$

$$Q_2 = 120\,\mu C \quad \textbf{(1/2)}$$

$$Q_2 = CV_2 \quad ...(ii)$$

By dividing Eq. (ii) from Eq. (i), we get

$$\frac{Q_1}{Q_2} = \frac{CV_1}{CV_2}$$

$$\Rightarrow \quad \frac{360}{120} = \frac{V}{V-120}$$

$$\Rightarrow \quad V = 180\text{ V} \quad \textbf{(1)}$$

$$\therefore \quad C = \frac{Q_1}{V_1} = \frac{360 \times 10^{-6}}{180}$$

$$= 2 \times 10^{-6}\text{ F} = 2\,\mu F$$

Hence, the potential, $V = 180$ V and unknown capacitance is $2\,\mu F$. **(1)**

(*ii*) Let Q be the charge stored in the capacitor

$$Q = CV$$

$$= 2 \times 10^{-6} \times 120$$

$$Q = 24 \times 10^{-5}\text{C} \quad \textbf{(1/2)}$$

24.A capacitor of 200 pF is charged by a 300 V battery. The battery is then disconnected and the charged capacitor is connected to another uncharged capacitor of 100 pF. Calculate the difference between the final energy stored in the combined system and the initial energy stored in the single capacitor.[Foreign 2012]

Ans.

Given, $C = 200$ pF $= 200 \times 10^{-12}$ F and $V = 300$ V

The energy (initially) stored by the capacitor is

$$U_i = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times 200 \times 10^{-12} \times 300 \times 300$$

$$= 9 \times 10^{-6}\text{ J}$$

The charge on the capacitor when charge through 300 V battery is

$$Q = CV$$
$$= 200 \times 10^{-12} \times 300$$
$$= 6 \times 10^{-8} C \quad \textbf{(1)}$$

When two capacitors are connected, they have their positive plates at the same potential and negative plates also at the same potential. Let V be the common potential difference. By charge conservation, charge would distribute but total charge would remains constant.

Thus, $Q = q + q'$

$$\frac{q}{C} = \frac{q'}{C'}$$

$$\frac{q}{200} = \frac{q'}{100}$$

$$q = 2q' \quad \textbf{(1)}$$

Thus, $Q = 2q' + q' = 3q'$

So, $q' = \frac{Q}{3} = \frac{60 \text{ nC}}{3}$

$= 20 \text{ nC}$

and $q = 2q' = 40 \text{ nC}$

Thus, final energy

$$U_f = \frac{q^2}{2C} + \frac{q'^2}{2C'}$$

$$= \frac{1}{2} \times \frac{(40 \times 10^{-9})^2}{200 \times 10^{-12}} + \frac{1}{2} \times \frac{(20 \times 10^{-9})^2}{100 \times 10^{-12}}$$

$$= 4 \times 10^{-6} + 2 \times 10^{-6},$$

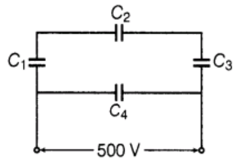
$$= 6 \times 10^{-6} \text{ J}$$

Difference in energy

$$= \text{final energy} - \text{initial energy}$$
$$= U_f - U_i$$
$$= 6 \times 10^{-6} - 9 \times 10^{-6}$$
$$= -3 \times 10^{-6} \quad \textbf{(1)}$$

Thus, difference in energy is -3×10^{-6} J.

25. A network of four capacitors each of 12 μF capacitance if connected to a 500 V supply as shown in the figure. Determine



(*i*) the equivalent capacitance of the network and

(*ii*) the charge on each capacitor.

[HOTS; All India 2010]

Ans.

In a series combination, where there is no division of charge,

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

In a parallel combination, where potential difference is same.

$$C_p = C_1 + C_2 + C_3 \ldots$$

(*i*) Here, C_1, C_2 and C_3 are in series; therefore their equivalent capacitance

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$C' = \frac{C}{3} = \frac{12}{3} = 4\,\mu\text{F}$$ **(1)**

Now, C' and C are in parallel combination.

$$\therefore \quad C_{net} = C' + C$$

$$= 4\,\mu\text{F} + 12\,\mu\text{F} = 16\,\mu\text{F}$$

$$C_{net} = 16\,\mu\text{F}$$ **(1)**

(*ii*) Being C' and C in parallel, 500 V potential difference is applied across them.

$\therefore$ Charge on C'

$$q_1 = C'V$$

$$= (4\,\mu\text{F}) \times 500$$

$$= 2000\,\mu\text{C}$$

$\therefore$ C_1, C_2 and C_3 capacitors each will have $2000\,\mu\text{C}$ charge.

Charge on C_4, $q_2 = C \times V$

$$= 12 \times 500$$

$$= 6000\,\mu\text{C}$$ **(1)**

26.A parallel plate capacitor is charged by a battery. After sometime, the battery is disconnected and a dielectric slab with its thickness equal to the plate separation is inserted between the plates. How will

(i)the capacitances of the capacitor,

(ii)potential difference between the plates and

(iii)the energy stored in the capacitors be affected? Justify your answer in each case.[Delhi 2010]

Ans.

On introduction of dielectric slab in a isolated charged capacitor.

(i) The capacitance (C') becomes K times of original capacitor as

$$C = \frac{\varepsilon_0 A}{d}$$

and $$C' = \frac{K \varepsilon_0 A}{d}$$ **(1)**

(ii) $\because$ Charge remains conserved in the phenomenon.

$$\therefore \quad CV = C'V'$$

$$V' = \frac{CV}{C'} = \frac{CV}{KC} \quad \text{[refer part (i)]}$$

$$\Rightarrow \quad V' = \frac{V}{K}$$ **(1)**

Potential difference decreases and become $\frac{1}{K}$ times of original value.

(iii) Energy stored initially,

$$U = \frac{q^2}{2C}$$

Energy stored later,

$$\therefore \quad U' = \frac{q^2}{2(KC)} \quad [\because C' = KC]$$

where, K = dielectric constant of medium

$$\Rightarrow \quad U' = \frac{1}{K}\left(\frac{q^2}{2C}\right)$$

$$\Rightarrow \quad U' = \frac{1}{K}(U)$$

$$U' = \frac{1}{K} \times U$$

The energy stored in the capacitor decreases and becomes $\frac{1}{K}$ times of original energy. **(1)**

27.A parallel plate capacitor, each with plate area A and separation d is charged to a potential difference V. The battery used to charge it remains connected. A dielectric slab of thickness d and dielectric constant K is now placed between the plates. What change if any will take place in

(i)charge on plates?

(ii)electric field intensity between the plates?

(iii)capacitance of the capacitor? Justify your answer in each case.[Delhi 2010]

Ans.

On introduction of dielectric slab to fill the gap between plates of capacitor completely when capacitor is connected with battery.

(*i*) The potential difference V between capacitors is same due to connectivity with battery and hence, charge q' becomes K times of original charge as

$q' = C'V' = (KC)(V) = K(CV) = Kq$

$q' = Kq$ **(1)**

(*ii*) Electric field intensity continue to be the same as potential difference and separation between two plates remain unaffected as

$$E = \frac{V}{d}.$$ **(1)**

(*iii*) The capacitance of capacitor becomes K times of original capacitor.

$$\therefore \quad C' = KC = \frac{K\varepsilon_0 A}{d}.$$ **(1)**

28.A parallel plate capacitor is charged to a potential difference V by a DC source. The capacitor is then disconnected from the source. If the distance between the plates is doubled, state with reason, how the following will change?

(i)Electric field between the plates

(ii)Capacitance

(iii)Energy stored in the capacitor [Delhi 2010]

Ans.

After disconnection from battery and doubling the separation between two plates

(*i*) Charge on capacitor remains same.

i.e. $CV = C'V'$

$$\Rightarrow \quad CV = \left(\frac{C}{2}\right)V' \quad \Rightarrow \quad V' = 2V$$

∵ Electric field between the plates

$$E' = \frac{V'}{d'} = \frac{2V}{2d}$$

$$E' = \frac{V}{d} = E$$

⇒ Electric field between the two plates remains same. **(1)**

(*ii*) Capacitance reduces to half of original value as

$$C \propto \frac{1}{d} \Rightarrow C' = \frac{C}{2}$$ **(1)**

(*iii*) Energy stored in the capacitor before disconnection from battery

$$U_1 = \frac{q^2}{2C}$$

Now, energy stored in the capacitor after disconnection from battery

$$U_2 = \frac{q^2}{2(C')} = \frac{q^2}{2 \times \left(\frac{C}{2}\right)} = \frac{q^2}{C}$$

$$\Rightarrow \quad U_2 = 2\left(\frac{q^2}{2C}\right) = 2U_1$$

$$U_2 = 2U_1$$

Energy stored in capacitor gets doubled to its initial value. **(1)**

29.Find the ratio of the potential differences that must be applied across the parallel and the series combination of two identical capacitors so that the energy stored in the two cases becomes the same.

[Foreign 2010]

Ans.

Let V_1 and V_2 are the potential differences across the series and parallel combination of two identical capacitors each of capacitance C.

Equivalent capacitance in series combination

$$C_s = \frac{C}{2}$$ **(1)**

Equivalent capacitance in parallel combination

$$C_p = 2C$$

According to the question,

$$U_s = U_p$$ **(1)**

$$\frac{1}{2}C_sV_s^2 = \frac{1}{2}C_pV_p^2$$

$$\Rightarrow \quad \frac{V_s^2}{V_p^2} = \frac{C_p}{C_s} = \frac{2C}{\left(\frac{C}{2}\right)}$$

$$\frac{V_s^2}{V_p^2} = 4 \Rightarrow \frac{V_s}{V_p} = 2$$

$$V_s : V_p = 2 : 1$$

30.(i) How is the electric field due to a charged parallel plate capacitor affected when a dielectric slab is inserted between the plates fully occupying the intervening region?

(ii) A slab of material of dielectric constant K has the same area as the plates of a parallel plate capacitor but has thickness 1/2 d,

where d is the separation between the plates. Find the expression for the capacitance when the slab is inserted between the plates. [Foreign 2010]

Ans.

(*i*) The total charge on the capacitor remains conserved on introduction of dielectric slab. Also, the capacitance of capacitor increases to K times of original values.

$\therefore \quad CV = C'V'$

$CV = (KC)V'$

$\Rightarrow \quad V' = \frac{V}{K}$

$\therefore$ New electric field,

$$E' = \frac{V'}{d} = \left(\frac{V/K}{d}\right) = \left(\frac{V}{d}\right)\frac{1}{K} = \frac{E}{K}$$

$\therefore$ On introduction of dielectric medium new electric field E' becomes $\frac{1}{K}$ times of its original value. **(1)**

(*ii*) $\because$ Capacitance of a parallel plate capacitor partially filled with dielectric medium is given by

$$C = \frac{\varepsilon_0 A}{(d - t + t/K)}$$ **(1)**

where, t is the thickness of dielectric medium.

Here, $t = \frac{d}{2}$

$$C = \frac{\varepsilon_0 A}{d - \frac{d}{2} + \frac{d}{2K}} = \frac{\varepsilon_0 A}{\frac{d}{2}\left(1 + \frac{1}{K}\right)}$$

$$\therefore \quad C = \frac{2\varepsilon_0 AK}{(K+1)d}$$ **(1)**

31.(i) Plot a graph comparing the variation of potential V and electric field E due to a point charge 0 as a function of distance R from the point charge.

(ii) Find the ratio of the potential differences that must be applied across the parallel and the series

combination of two capacitors, C_l and C_2 with their capacitances in the ratio 1 : 2, so that the energy stored in the two cases becomes the same[Foreign 2010]

Ans.

(*i*) The graph comparing the variation of potential V and electric field. Refer to ans. 20 Topic -1 **(1)**

(*ii*) Let $C_1 = C$ and $C_2 = 2C$

$\therefore$ Equivalent capacitance

In series, $\quad C_s = \frac{2C \times C}{2C + C} = \frac{2C}{3}$

In parallel, $C_p = 2C + C = 3C$ **(1)**

$\therefore$ V_p and V_s are potential difference across the final capacitor in parallel and series combination respectively, to have same potential energy.

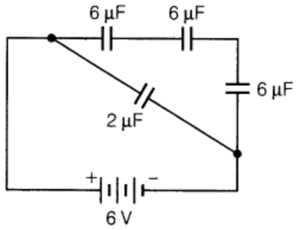
$$U_p = U_s$$

$$\frac{1}{2}C_pV_p^2 = \frac{1}{2}C_sV_s^2$$

$$\frac{V_p}{V_s} = \sqrt{\frac{C_s}{C_p}} = \sqrt{\frac{(2C/3)}{(3C)}} = \sqrt{\frac{2}{9}}$$

$$V_p : V_s = \sqrt{2} : 3$$ **(1)**

32. Four capacitors of values $6\,\mu F, 6\,\mu F, 6\,\mu F$ and $2\,\mu F$ are connected to a 6 V battery as shown in the figure. Determine the



(*i*) Equivalent capacitance of the network.

(*ii*) The charge on each capacitor.

[Delhi 2010C]

Ans.

In series combination, charge on each capacitor is the same.

(*i*) All capacitors of $6\,\mu F$ are in series combination, then equivalent capacitance is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

or $$C' = \frac{C}{n} = \frac{6\,\mu F}{3} = 2\,\mu F$$

C' and $2\,\mu F$ capacitors are in parallel combination.

∴ Equivalent capacitance

$$C_{eq} = C' + 2\,\mu F$$
$$= 2\,\mu F + 2\,\mu F$$
$$C_{eq} = 4\,\mu F$$ **(1)**

(*ii*) Since, C' and $2\,\mu F$ are in parallel combination, therefore , same potential difference 6 V is applied on them.

∴ Charge on C'

$$q' = C'V = (2\,\mu F) \times 6\text{ V} = 12\,\mu C$$

The charge across the each capacitor of $6\,\mu F$ capacitor is same and equal to charge across the combination i.e. $12\,\mu C$.

Charge on $2\,\mu F$ capacitor

$$q = CV = (2\,\mu F)\,(6\text{ V}) = 12\,\mu C$$

33.A parallel plate* capacitor is charged by a battery. After sometime, the battery is disconnected and a dielectric slab of dielectric constant K is inserted between the plates. How would
(i)the electric field between the plates
(ii)the energy stored in the capacitor be affected? Justify your answer. [All India 2009]

Ans.(i) The total charge on the capacitor remains conserved on introduction of dielectric slab. Also, the capacitance of capacitor increases to K times of original values.

(*ii*) Energy stored in capacitor initially,

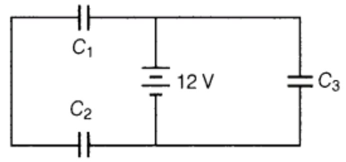
$$U = \frac{1}{2}CV^2$$

(Energy stored in capacitor later,

$$U' = \frac{1}{2}C'V'^2 = \frac{1}{2}(KC)\left(\frac{V}{K}\right)^2$$

$$U' = \frac{1}{K}\left(\frac{1}{2}CV^2\right) = \frac{U}{K}\,U' = \frac{U}{K}$$

34. Three identical capacitors C_1, C_2 and C_3 of capacitances 6 μF each are connected to a 12 V battery as shown below:



Find

(*i*) the charge on each capacitor

(*ii*) the equivalent capacitances of the network

(*iii*) the energy stored in the network of capacitors

[Delhi 2009C]

Ans.

(*i*) The equivalent capacitance of C_1 and C_2 connected in series

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \quad C' = \frac{6}{2} = 3\,\mu F$$

$\therefore$ Charge, $q' = C'V = (3\,\mu F)\,12 = 36\,\mu C$

$\therefore$ Charge on each capacitor of C_1 and C_2 is 36 μC.

Charge on C_3,

$$q_3 = C_3 V = (6\,\mu F) \times 12 = 72\,\mu C$$ **(1)**

$$q_3 = 72\,\mu C$$

(*ii*) Equivalent capacitance of network,

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} + C_3$$

$$= \frac{6 \times 6}{6 + 6} + 6$$

$$= 3 + 6 = 9\,\mu F$$

$$C_{eq} = 9\,\mu F$$ **(1)**

(*iii*) Energy stored in the capacitor

$$U = U_1 + U_2 + U_3$$

$$= \frac{q'^2}{2C_1} + \frac{q'^2}{2C_2} + \frac{q^2}{2C_3}$$

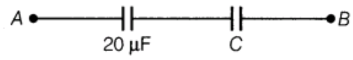
$\because \; C_1 = C_2 = C_3 = 6\,\mu F$

$$\therefore \; U = \frac{1}{(12\,\mu F)}[q'^2 + q'^2 + q^2]$$

$$= \frac{1}{(12\,\mu F)}[(36\,\mu C)^2 + (36\,\mu C)^2 + (72\,\mu C)^2]$$

$$U = 648\,\mu J$$ **(1)**

35. The equivalent capacitance of the combination between points A and B in the given figure is 4 μF.



(*i*) Calculate the capacitance of the capacitor C.

(*ii*) Calculate the charge on each capacitor if a 12 V battery is connected across terminals A and B.

(*iii*) What will be the potential drop across each capacitor?

[Delhi 2009]

Ans.

According to the question,

Capacitors of 20 μF and C are connected in series.

(*i*) The equivalent capacitance

$$(4\,\mu F) = \frac{(20\,\mu F)\times C}{20 + C}$$

$$(20 + C) = 5C$$

$$4C = 20 \quad \therefore \quad C = 5\,\mu F$$ **(1)**

(*ii*) Charge on capacitor (equivalent)

$$q = (4\,\mu F)\times 12 = 48\,\mu C$$

same charge 48 μC lies on both the capacitors. **(1)**

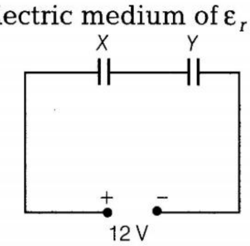
(*iii*) Potential drop across 20 μF capacitor

$$V_1 = \frac{q}{C_1} = \frac{48\,\mu C}{20\,\mu F} = 2.4\text{ V}$$

Potential drop across 5 μF capacitor

$$V_2 = \frac{q}{C_2} = \frac{48\,\mu C}{5\,\mu F} = 9.6\text{ V} \quad \left(\frac{1}{2}\times 2 = 1\right)$$

36. Two parallel plate capacitors X and Y have the same area of the plates and same separation between them. X has air between the plates while Y contains a dielectric medium of $\varepsilon_r = 4$.



(*i*) Calculate the capacitance of each capacitor if equivalent of the combination is 4 μF.

(*ii*) Calculate the potential difference between the plates of X and Y.

(*iii*) What is the ratio of electrostatic energy stored in X and Y? **[Delhi 2009]**

Ans.

A dielectric medium is inserted between the plates of a condenser in place of air, its capacity becomes K times of original one.

The capacitance of two capacitors are

$$C_x = \frac{\varepsilon_0 A}{d} = C \quad \text{[say]}$$

$$C_y = \frac{4\varepsilon_0 A}{d} = 4C$$

(*i*) According to the problem,

$$C_{eq} = 4\,\mu F$$

$$\therefore \quad C_{eq} = \frac{C_x \times C_y}{C_x + C_y}$$

$$4\,\mu F = \frac{C \times 4C}{C + 4C} = \frac{4C}{5}$$

$$\left[\because \text{For series combination, } C = \frac{C_1 C_2}{C_1 + C_2}\right]$$

$$C = 5\,\mu F$$

$$\Rightarrow \quad C_x = C = 5\,\mu F$$

$$C_y = 4C = 4 \times 5\,\mu F = 20\,\mu F$$ **(1/2 × 2 = 1)**

(*ii*) $\because$ Charge across the combination

$$q = C_{eq} V$$

$$= (4\,\mu F) \times 12$$

$$= 48\,\mu C$$

$\therefore$ Same charge, i.e. 48 μC lies on each capacitor being in series combination.

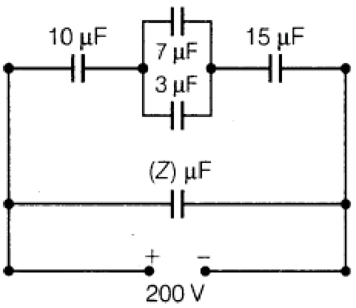
$\therefore$ Potential difference across C_x,

$$V_1 = \frac{q}{C_x} = \frac{48\,\mu C}{5\,\mu F} = 9.6\text{ V}$$

Potential difference across C_y,

$$V_2 = \frac{q}{C_y} = \frac{48\,\mu C}{20\,\mu F} = 2.4\text{ V}$$ **(1/2 × 2 = 1)**

37.A system of capacitors connected as shown in the figure has a total energy of 160 mJ stored in it. Obtain the value of the equivalent capacitance of this system and the value of Z. [All India 2009 c]



Ans.

Given, total energy $=160\,\text{mJ}$

Let equivalent capacitance of the combination of capacitors is C.

According to the question,

$$\frac{1}{2}CV^2 = 160 \times 10^{-3}\,\text{J}$$

$$\Rightarrow \frac{1}{2} \times C \times (200)^2 = 160 \times 10^{-3}$$

$\Rightarrow$ Equivalent capacitance $C = 8\,\mu\text{F}$ **(1)**

$\because$ Equivalent capacitor of $7\,\mu\text{F}$ and $3\,\mu\text{F}$ connected in parallel $= 7 + 3 = 10\,\mu\text{F}$

$\therefore$ $10\,\mu\text{F}, 10\,\mu\text{F}$ [combination of $7\,\mu\text{F}$ and $3\,\mu\text{F}$] and $15\,\mu\text{F}$ are in series combination. Therefore, their equivalent capacitance

$$\frac{1}{C'} = \frac{1}{10} + \frac{1}{10} + \frac{1}{15} = \frac{3+3+2}{30} = \frac{8}{30}$$

$$C' = \frac{15}{4}\,\mu\text{F}$$ **(1)**

But, C' and Z are in parallel combination, therefore equivalent capacitance

$$\frac{15}{4} + Z = C \qquad \text{[from Eq.(i)]}$$

$$\frac{15}{4} + Z = 8$$

$$Z = 8 - \frac{15}{4} = \frac{32-15}{4}$$

$$Z = \frac{17}{4}\,\mu\text{F}$$

$$Z = 4.25\,\mu\text{F}$$ **(1)**

38.The two plates of a parallel plate capacitor are 4 mm apart. A slab of dielectric constant 3 and thickness 3 mm is introduced between the plates with its faces parallel to them. The distance between the plates is so adjusted that the capacitance of the capacitor becomes (2/3)rd of its original value. What is the new distance between the plates?[All India 2008 C]

Ans.

Before introduction of slab,

$$d_1 = 4\,\text{mm} = 4 \times 10^{-3}\ \text{m}$$

Plate area = A [say]

$\therefore$ Capacitance, $C_1 = \dfrac{\varepsilon_0 A}{d_1}$

Suppose after introduction of slab, the new distance between the place is d_2.

Plate area of each plate = A

$$K = 3$$

Thickness of slab,

$$t = 3\,\text{mm} = 3 \times 10^{-3}\ \text{m}$$

$$\text{Capacitance, } C_2 = \frac{\varepsilon_0 A}{(d_2 - t + t/K)}$$ **(1)**

According to the question,

$$\frac{2}{3} C_1 = C_2$$

$$\frac{2}{3} \cdot \frac{\varepsilon_0 A}{d_1} = \frac{\varepsilon_0 A}{d_2 - t + t/K}$$

Substituting the values [given in question]

$$\frac{2}{3d_1} = \frac{1}{d_2 - t + t/K}$$ **(1)**

$$\frac{2}{3 \times 4 \times 10^{-3}} = \frac{1}{(d_2 - 3 + 3/3) \times 10^{-3}}$$

[d_2 is in mm]

$$\frac{1}{6} = \frac{1}{(d_2 - 2)}$$

$\Rightarrow \quad d_2 - 2 = 6$

$\Rightarrow \quad d_2 = 8\ \text{mm}$ **(1)**

5 Marks Questions

39.(i) A parallel plate capacitor is charged by a battery to a potential. The battery is disconnected and a dielectric slab is inserted to completely fill the space between the plates. How will

(a)its capacitance

(b)electric field between the plates and

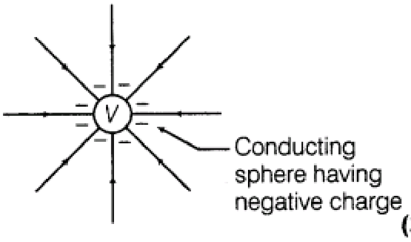
(c)energy stored in the capacitor be affected? Justify your answer giving necessary mathematical expressions for each case.

(ii) (a) Draw the electric field lines due to a conducting sphere.

(b) Draw the electric field lines due to a dipole.

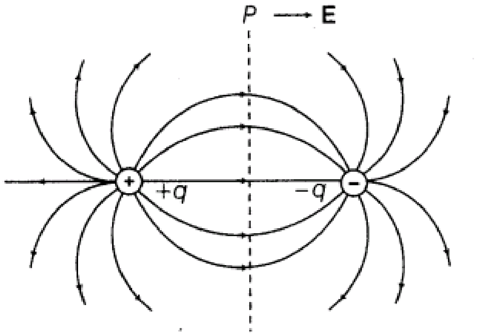
Ans.

(ii) (a) Electric field lines due to a conducting sphere are shown in the figure.



(2)

(b) Electric field lines due to an electric dipole are shown in the figure



(1)

40. (*i*) Deduce the expression for the energy stored in a charged capacitor.

(*ii*) Show that the effective capacitances C of a series combination of three capacitors C_1, C_2 and C_3 is given by

$$C = \frac{C_1 C_2 C_3}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$

[HOTS, All India 2010C]

Ans.

> As, charge on capacitor increases we have to work more against electrostatic repulsion and this amount of work done will be stored as potential energy in the capacitor.

(*i*) We know that,

$$q = CV \Rightarrow V = q/C$$

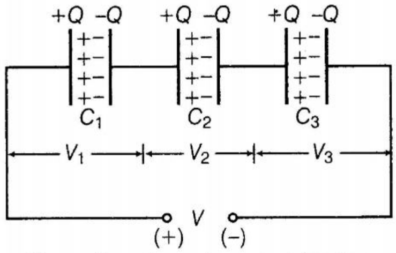
$$dW = Vdq = \frac{q}{C} dq$$

where, q = instantaneous charge,
C = instantaneous capacitance and
V = instantaneous voltage

∴ Total work done in storing charge from 0 to q is given by

$$W = \int_0^q \frac{q}{C} dq = \frac{q^2}{2C}$$ **(2)**

(*ii*) In series combination of capacitors, same charge lie on each capacitor for any value of capacitances.



Capacitors in series combination

(1)

Also, potential difference across the combination is equal to the algebraic sum of potential differences across each capacitor.

i.e. $V = V_1 + V_2 + V_3$ ...(i)

where, V_1, V_2, V_3 and V are the potential differences across C_1, C_2, C_3 and equivalent capacitor, respectively.

$$\therefore \quad q = C_1V_1 \quad \Rightarrow \quad V_1 = \frac{q}{C_1}$$

Similarly, $V_2 = \frac{q}{C_2}$

$$V_3 = \frac{q}{C_3} \quad \text{[from Eq. (i)]}$$

∴ Total potential difference

$$V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} \quad \textbf{(1)}$$

$$\frac{V}{q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

[$\because$ $\frac{V}{q} = \frac{1}{C}$, where C is equivalent capacitance of combination]

or $$\frac{1}{C} = \frac{C_2 C_3 + C_3 C_1 + C_1 C_2}{C_1 C_2 C_3}$$

$$\Rightarrow \quad C = \frac{C_1 C_2 C_3}{C_1 C_2 + C_2 C_3 + C_3 C_1} \quad \textbf{(1)}$$

41.(i)Show that in a parallel plate capacitor if the medium between the plates of a capacitor is filled with an insulating substance of dielectric constant K, its capacitance increases.

(ii) Deduce the expression for the energy stored in a capacitor of capacitance C with charge Q.[Delhi 2009 C]

Ans.

On introduction of dielectric slab in a isolated charged capacitor.

(*i*) The capacitance (C') becomes K times of original capacitor as

$$C = \frac{\varepsilon_0 A}{d}$$

and $$C' = \frac{K \varepsilon_0 A}{d} \quad \textbf{(1)}$$

(*ii*) $\because$ Charge remains conserved in the phenomenon.

$$\therefore \quad CV = C'V'$$

$$V' = \frac{CV}{C'} = \frac{CV}{KC} \quad \text{[refer part (i)]}$$

$$\Rightarrow \quad V' = \frac{V}{K} \quad \textbf{(1)}$$

Potential difference decreases and become $\frac{1}{K}$ times of original value.

(iii) Energy stored initially,

$$U = \frac{q^2}{2C}$$

Energy stored later,

$$\therefore \quad U' = \frac{q^2}{2(KC)} \qquad [\because C' = KC]$$

where, K = dielectric constant of medium

$$\Rightarrow \quad U' = \frac{1}{K}\left(\frac{q^2}{2C}\right)$$

$$\Rightarrow \quad U' = \frac{1}{K}(U)$$

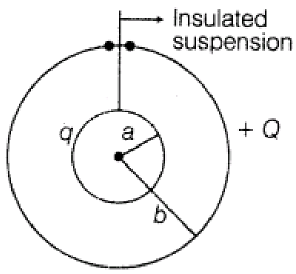
$$U' = \frac{1}{K} \times U$$

The energy stored in the capacitor decreases and becomes $\frac{1}{K}$ times of original energy. **(1)**

42.A small sphere of radius a carrying a positive charge q is placed concentrically inside a larger hollow conducting shell of radius b(b> a). This outer shell has charge Q on it. Show that if these spheres are connected by a conducting wire, charge will always flow from the inner sphere to the outer sphere irrespective of the magnitude of the two charges.

Ans.

(i) Let small sphere has charge q and radius a is placed inside a outer shell of charge $+Q$ and radius b.



Electric potential on the small sphere due to its own charge q.

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{a} \qquad \ldots\text{(i)}$$

where, q = charge on small sphere

a = radius of small sphere

Similarly, electric potential on outer sphere due to its own charge

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{b} \quad \text{...(ii)}$$

where, Q = charge of outer shell

b = radius of outer shell. **(1 × 2 = 2)**

Also, same potential V_2 exists at every point inside outer shell due to its own charge, $+Q$.

Now, net electric potential at inner sphere of radius a.

V_i = Electric potential due to its own charge

$$V_i = \frac{1}{4\pi\varepsilon_0} \frac{q}{a} + \frac{1}{4\pi\varepsilon_0} \frac{Q}{b} \quad \text{...(iii)}$$

Net electric potential at outer sphere due to charge on the both spheres

$$V_o = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{b} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{b} \quad \text{...(iv)}$$

$$\therefore \quad V_i - V_o = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) \quad \text{...(v)}$$

From Eqs. (iii) and (iv), we get

$\because \quad a < b, \frac{1}{a} > \frac{1}{b}$

$\Rightarrow \quad V_i - V_o > 0$ **(2)**

Thus, inner sphere has net potential higher than potential of outer sphere for every value of q and Q. Therefore, when they are connected by a wire, positive charge always flow from inner sphere (at higher potential) to outer sphere (at lower potential) irrespective of the magnitude of charge. **(1)**

43. (*i*) Derive an expression for the energy stored in a parallel plate capacitor.

(*ii*) On charging a parallel plate capacitor to a potential V, the spacing between the plates is halved and a dielectric medium of $\varepsilon_r = 10$ is introduced between the plates without disconnecting the DC source. Explain, using suitable expressions, how the

(*a*) capacitance

(*b*) electric field and

(*c*) energy density of the capacitor changes? **[All India 2008]**

Ans.(i) An expression for energy stored in a charged capacitor. (ii) Connected battery ensures same potential difference (V) across capacitor even after reducing the spacing between plates halved and introducing the dielectric slab.

(a) If initial capacitance,

$$C_1 = \frac{\varepsilon_0 A}{d}$$

Then, final capacitance

$$C_2 = \frac{10\varepsilon_0 A}{d/2}$$

$$= \frac{20\varepsilon_0 A}{d} = 20C_1$$

$$C_2 = 20C_1$$

Capacitance increases to 20 times of original value. **(1)**

(b) $\because$ Initial electric field, $E_1 = \frac{V}{d}$

Finally,

$$E_2 = \frac{V}{d/2} = 2\left(\frac{V}{d}\right) = 2E_1$$

$$E_2 = 2E_1$$

Electric field intensity gets doubled. **(1)**

(iii) Energy density of the capacitor

$$U_1 = \frac{1}{2}\varepsilon_0 E_1^2 = \frac{1}{2}\varepsilon_0 E_2^2$$

$$U_2 = \frac{1}{2}\varepsilon_0 (2E_1)^2$$

$$= \frac{1}{2}\varepsilon_0 (4E_1^2)$$

$$\Rightarrow U_2 = 4U_1$$ **(1)**

Energy density becomes 4 times of its original value.

44.Define the terms (i)capacitance of a capacitor (ii)dielectric strength of a dielectric (iii)When a dielectric is inserted between the plates of a charged parallel plate capacitor fully occupying the intervening region, how does the polarisation of the dielectric medium affect the net electric field? For a linear dielectric,show that the introduction of the dielectric increases its capacitance by a factor K which is a characteristic of the dielectric. [Delhi 2008 C]

Ans.

(*i*) For a given capacitor, the charge q on the capacitor is proportional to the potential difference V between the plates.

i.e. $q \propto V$

$q = CV$ **(1)**

The proportionality constant, C is called the capacitance of the capacitor.

It depends on the shape, size and geometry of capacitor. **(1)**

(*ii*) **Dielectric strength** The dielectric strength of the dielectrics is equal to that maximum value of electric field that can exist in that dielectric without causing the dielectric breakdown of its insulating property. **(1)**

(*iii*) Other part of question refer to ans. 20(a) (topic-1) and on introduction of dielectric between the plates of capacitor.

As $E_p \propto E_0$

$\therefore$ $E_p = \frac{1}{k} E_0$

$\Rightarrow$ $V = Ed = \frac{Qd}{\varepsilon_0 AK} = \frac{V_0}{K}$

Thus, capacitance is

$$C = \frac{Q}{\frac{V_0}{K}} = \frac{KQ}{V_0} = KC_0 \Rightarrow \frac{C}{C_0} = K$$ **(1)**

K is a factor (>1) by which the capacitance is increased. **(1)**